Sep 27-8:59 AM

3.7b Implicit Differentation

Show that dy/dx is defined at every point on the graph of $2y = x^2 + \sin(y)$

$2y' = 2x + y'\cos y$

$2y' - y'\cos y = 2x$

$y'(2 - \cos y) = 2x$

$y' = \frac{2x}{2-\cos y}$

$2 - \cos y = 0$

$\cos y = 2$

never undef.

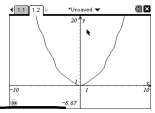
$-1 \le \cos y \le 1$

so always defined

Graph the curve using parametric equations

$x^2 = 2y - \sin y$

$x = \pm\sqrt{2y - \sin y}$



$x = \sqrt{2t - \sin t}$

$y = t$

Sep 29-7:40 AM

$x^2 - 2xy + y^2 = 4$

1. Find dy/dx

$2x - (2xy' + 2y) + 2yy' = 0$

$2x - 2xy' - 2y + 2y \cdot y' = 0$

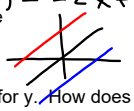
$-2xy' + 2yy' = -2x + 2y$

$y'(-2x + 2y) = -2x + 2y$

$y' = \frac{-2x+2y}{-2x+2y}$

$y' = 1$

2. Use dy/dx to sketch a possible graph of the implicit curve.



3. Factor the left side and solve for y. How does this compare with your graph?

$\sqrt{(x-y)^2} = \sqrt{4}$

$|x - y| = 2$

$x - y = \pm 2$

$y = x \mp 2$

Sep 29-8:00 AM

Prove the power rule for rational exponents

$\frac{d}{dx} x^n = n x^{n-1}$

$\frac{d}{dx} x^{\frac{p}{q}} = \frac{p}{q} x^{\frac{p}{q}-1}$

$(y)^q = (x^{p/q})^q$

$y^q = x^p$

$qy^{q-1} \cdot y' = px^{p-1}$

$y' = \frac{px^{p-1}}{qy^{q-1}} = \frac{p}{q} \frac{x^{p-1}}{(x^{\frac{p}{q}})^{q-1}}$

$y' = \frac{p}{q} \frac{x^{p-1}}{x^{\frac{p}{q}(q-1)}}$

$y' = \frac{p}{q} \frac{x^{p-1}}{x^{p-\frac{p}{q}}}$

$= \frac{p}{q} x^{p-1-(p-\frac{p}{q})}$

$\frac{p}{q} x^{\frac{p}{q}-1}$

$\frac{d}{dx}\sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}}$

$= \frac{1}{2} x^{-\frac{1}{2}}$

$\frac{d}{dx}\sqrt[4]{(x-1)^3}$

$\frac{d}{dx}(x-1)^{\frac{3}{4}}$

$= \frac{3}{4}(x-1)^{-\frac{1}{4}}$

$= \frac{3}{4} \frac{1}{(x-1)^{1/4}}$

$= \frac{3}{4\sqrt[4]{x-1}}$

Sep 29-8:12 AM

Find the slope of the Folium of Descartes at the points (4,2) and (2,4).

$x^3 + y^3 - 9xy = 0$

$3x^2 + 3y^2 y' - (9xy' + 9y) = 0$

$3x^2 + 3y^2 y' - 9xy' - 9y = 0$

$3y^2 y' - 9xy' = -3x^2 + 9y$

$y'(3y^2 - 9x) = -3x^2 + 9y$

$y' = \frac{-3x^2 + 9y}{3y^2 - 9x}$

$y'(4,2) = \frac{5}{4}$

$y'(2,4) = \frac{4}{5}$

Find the points where the folium has: (a) a horizontal tangent;
(b) a vertical tangent

a) horiz $y' = 0$ $-3x^2 + 9y = 0$ $y = \frac{3x^2}{9}$

b) vert $y' = \not{*}$ $3y^2 - 9x = 0$ $x = \frac{3y^2}{9}$

Sep 29-8:15 AM